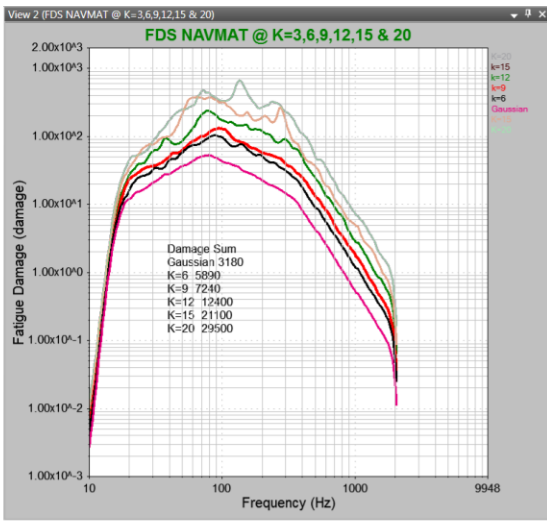
Σ_{FDS} -- Kurtosion Correlation

The Damage Sum or $\Sigma_{FDS,}$ has been defined as the sum of all 1/24th Octave SDoF breakpoints of the fatigue damage spectrum or FDS. As a broadband indicator, FDS is a relative, quantitative metric describing the severity of an excitation or the damage of an EUE. In narrowband use, with known UUT failure modes, material and resonance qualities, FDS is used to re-create field failures and accelerate long duration tests.

FDS can be used to generate an equivalent damage test profile from one or more imported time histories to characterize a product life, warranty, document a HALT/HASS step stress process, correlate HALT to HASS and more relevantly, generate reliability estimates and demonstration tests.

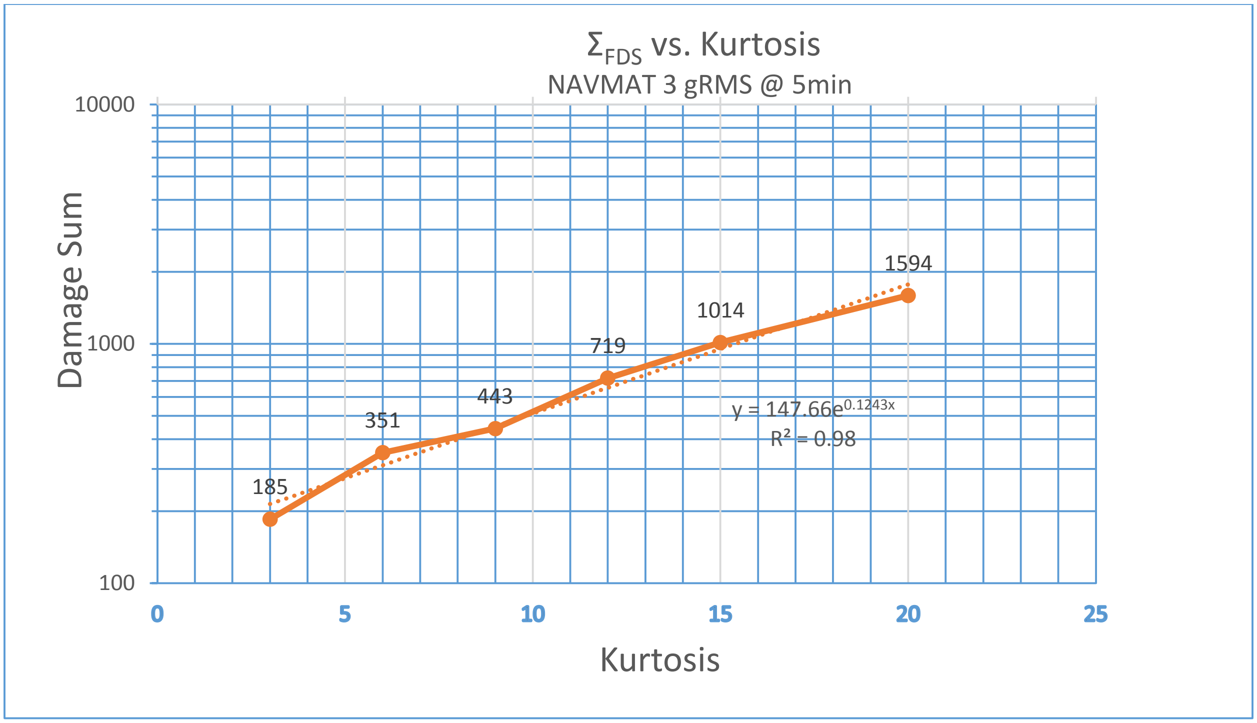


The companion tool to the FDS is Vibration Research's patented Kurtosion©, which captures the damage from all excitations, non-Gaussian and non-stationary excitations, in contrast to these short-comings of traditional FFT-based PSD and gRMS tools.

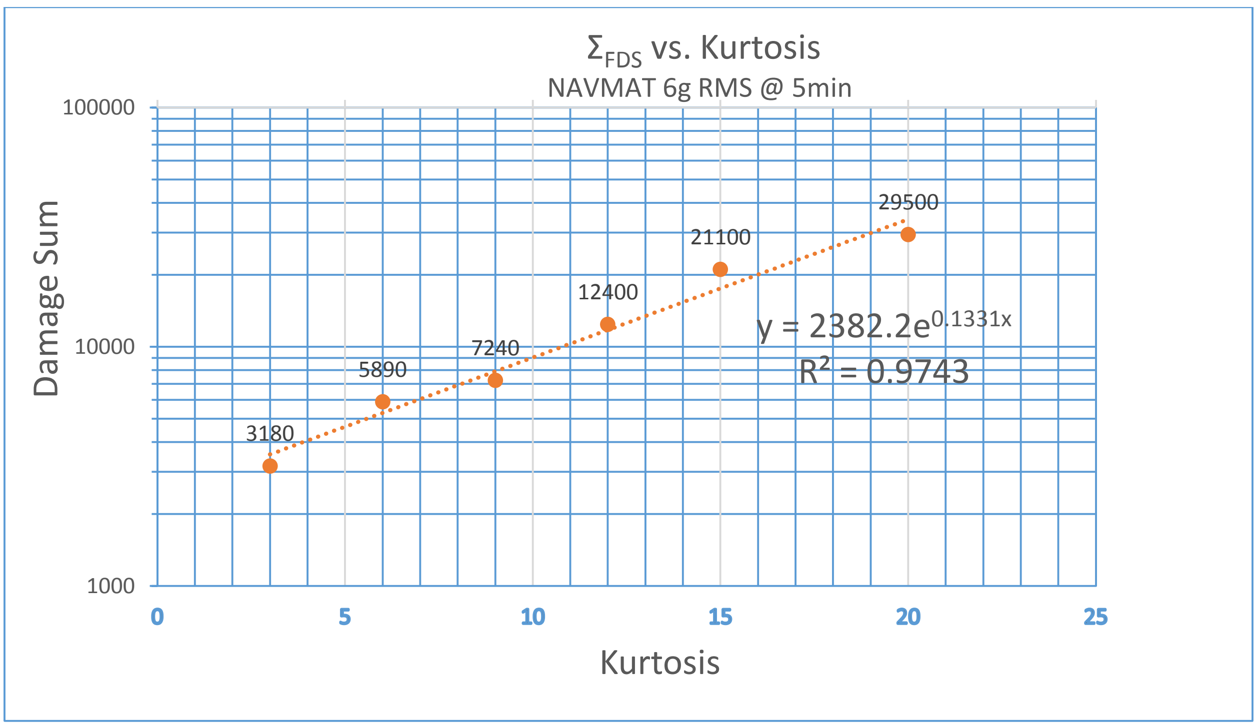
An exercise to show the effect the FDS of increasing kurtosis was run on the NAVMAT P9492 haystack PSD @ 6.06 gRMS. Damage Sums were calculated and shown.

Additional time histories were generated to investigate any correlation between the statistically more-accurate Kurtosion describing actual PPDs of EUE time histories, and the FDS, which incorporates a velocity-based rainflow-counted cycles and a Miner's Rule exponential weighting of stress to fatigue.

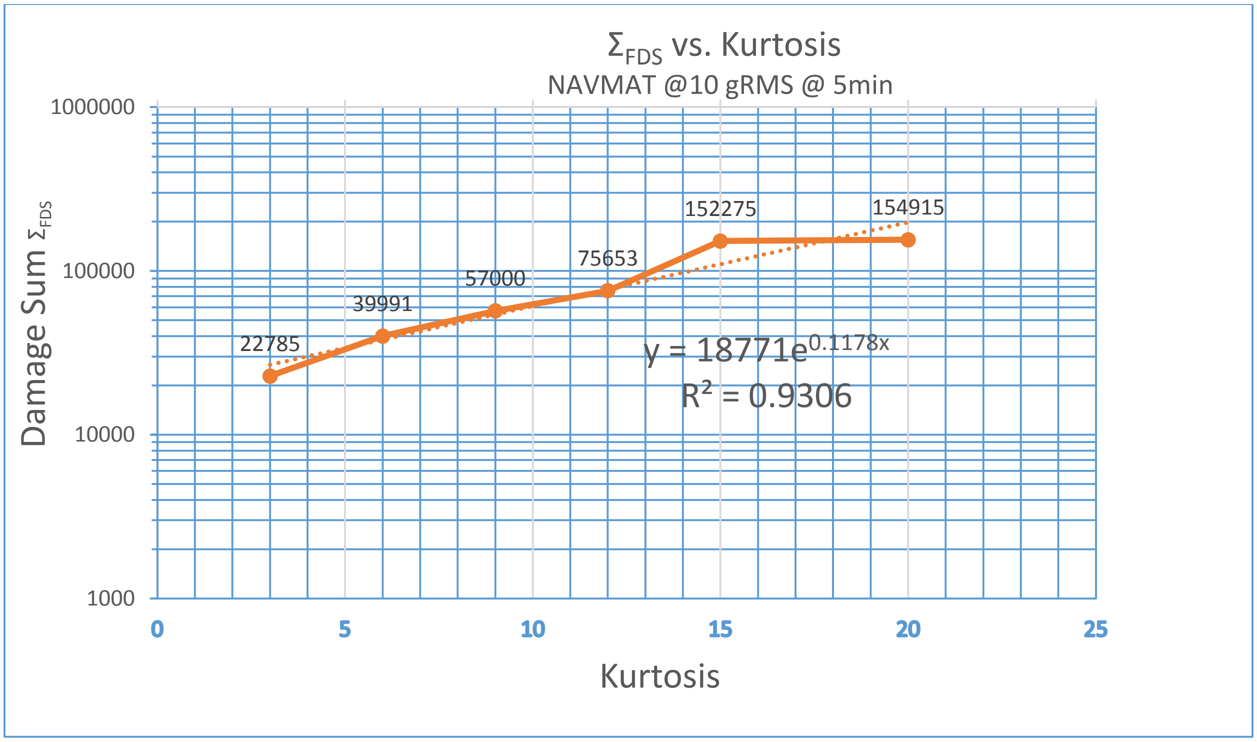
Consistent in the process of generating the following was the use of the 1979 NAVMAT PSD @ 6.06 gRMS. Only the kurtosis values of Gaussian (3), 6, 9, 12, 15 & 20 were varied and applied to generate the equal 5 minute recorded time histories. Because the FFT-based PSD does not accommodate non-Gaussian and non-stationary waveforms, the gRMS remained unchanged but the Σ_{FDS} captures the damage.

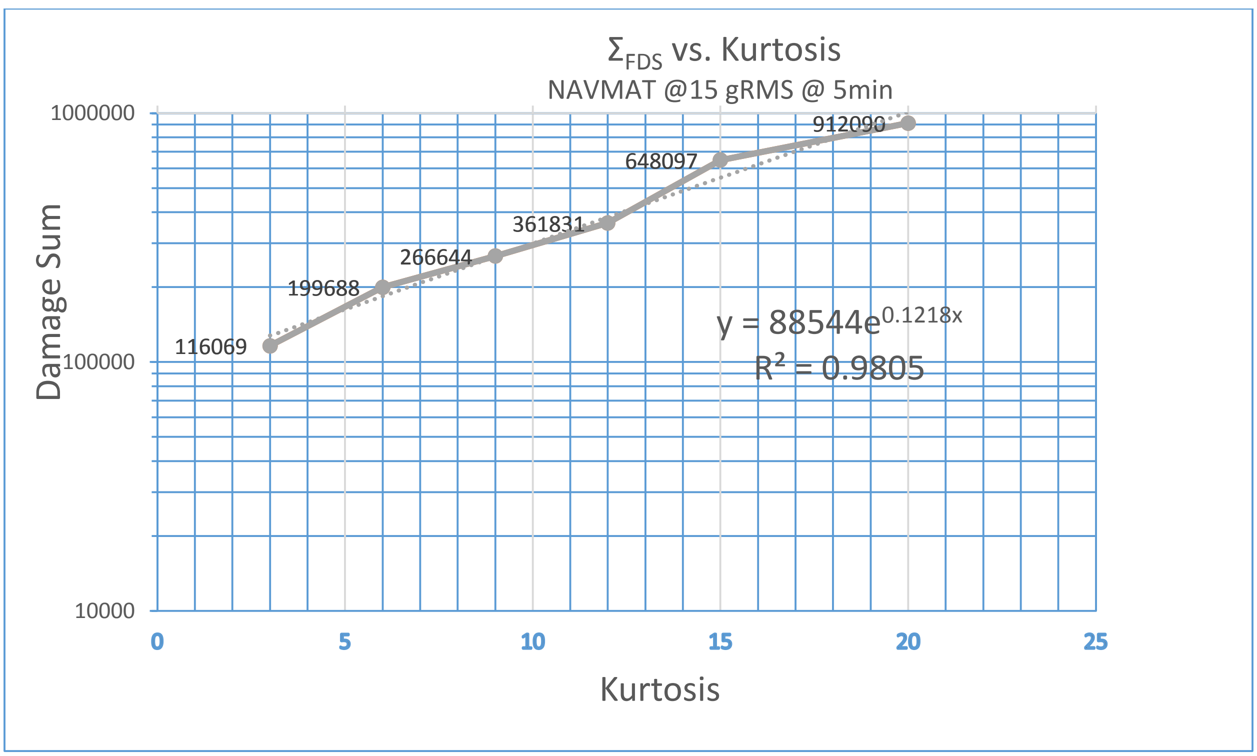


Σ_{FDS} vs. Kurtosis
NAVMAT 3 gRMS @ 5min
Damage Sum
Kurtosis
10000
1000
100
0
5
10
15
20
25
185
351
443
719
1014
1594
$y = 147.66e^{0.1243x}$
$R^2 = 0.98$



Σ_{FDS} vs. Kurtosis
NAVMAT 6g RMS @ 5min
Damage Sum
Kurtosis
100000
10000
1000
0
5
10
15
20
25
3180
5890
7240
12400
21100
29500
$y = 2382.2e^{0.1331x}$
$R^2 = 0.9743$





What is shown in the above is the relevance of the Σ_{FDS} as a consistent measure of damage given as an exponential curve fit regression with high correlation to the Kurtosis of a time history.

The relationship of the intercept of the regression line at kurtosis equal to zero and correspondingly, to the gRMS power of the excitation has not been figured out. Perhaps to something else. What does now seem possible, however, is a statistical correlation and thus, possible predictive value using the more relevant metrics of kurtosis and Σ_{FDS} for some or any of the following:

- Time to equivalent damage for failures
  - Physical
  - Operational—Powered and monitored
- Derivation of a Damage Number*
- Design or reliability margins—model service life compare with HALT step stress results
- Application of ED shakers to
  - HALT/HASS
  - Margin testing
  - Reliability Demo and Growth
  - Spec requirements vs. EUE
  - Not worth doing for RS machines—damage not related to gRMS setpoints
- *Replace gRMS ratios with Σ_{FDS} ratios as the metric for test compression. Mil 810G, et al. No weighting exponent required because it is damage with weighting exponent incorporated.

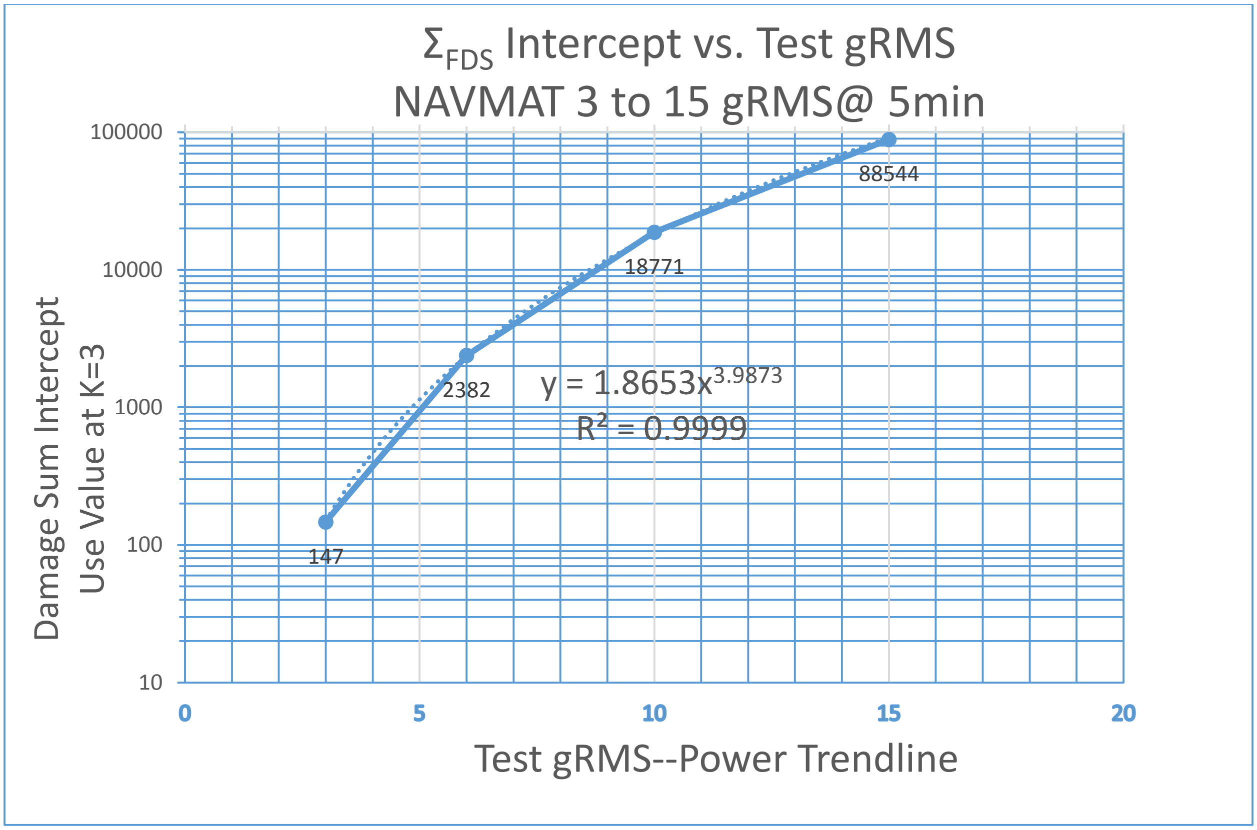
Cycles to failure isn't relevant for anything but sine. Attempting distill FDS from random (Gaussian or not) to cycles should be discussed. Time to failure for random tests based on FDS, which incorporates damage from high kurtosis is valuable, certainly within Tom Irvine's caveats:

"Vibration fatigue calculations are "ballpark" calculations given uncertainties in S-N curves, stress concentration factors, mean stress, non-linearity, temperature and other variables. Furthermore, the order of loading over a system's lifetime may affect the true fatigue life. Note that the Palmgren-Miner summation assumes that the damage mechanism is the same at higher stress levels as at lower ones."

I find one interpretation to mean FDS and Σ_{FDS} not be subject to any false precision, rather for the better-than-traditional PSD and gRMS metrics that they offer.

<u>Damage Sum Test Power Correlation</u>

Pursuing Σ_{FDS} correlations further, the following runs were made scaling the nominal 6 gRMS random both up and down from 3 gRMS to 15 gRMS. The goal was a relationship between the Σ_{FDS} and the gRMS power of a Gaussian PSD. The plot of the Σ_{FDS} intercepts from the above Σ_{FDS} vs. Kurtosis plots vs. the gRMS power of the Gaussian test PSDs. It yields a power function curve fit with very high correlation. The intercepts were used even though there is little or no meaning to a kurtosis of zero.



Given consistent correlation of Σ_{FDS} to Kurtosis at different gRMS test levels, the Σ_{FDS} intercept correlation to gRMS test levels is nearly perfect and also close to the 4^{th} power of the gRMS test level-- possibly related to kurtosis being the 4^{th} moment about the mean.

One could select a gRMS test level, say from a spec requirement, determine the Σ_{FDS} intercept, return to the Σ_{FDS} vs. Kurtosis plots and, using the exponential curve fit, estimate the cumulative fatigue damage for the test. Working the other way, an FDS representing one or more imported EUE time histories, scaled to represent a service life sets a target Σ_{FDS} which can be used to generate a Gaussian PSD of equivalent damage, and reintroduce Kurtosis to lower the gRMS power, shaker duty cycle and accelerate the test.

There is a strong argument that FDS and Σ_{FDS} become part of vibration test requirements for HALT/HASS, qualification, RDGT, and product strength, warranty requirements and reliability and confidence estimates.